

Black Holes and Large N Species Solution to the Hierarchy Problem

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Abstract

We provide the perturbative and non-perturbative arguments showing that theories with large number of species of the quantum fields, imply an inevitable hierarchy between the masses of the species and the Planck scale, shedding a different light on the hierarchy problem. In particular, using the black hole physics, we prove that any consistent theory that includes N Z_2 -conserved species of the quantum fields of mass Λ , must have a value of the Planck mass, which in large N limit is given by $M_P^2 \gtrsim N\Lambda^2$. An useful byproduct of this proof is that any exactly conserved quantum charge, not associated with a long-range classical field, must be defined maximum modulo N , with $N \lesssim (M_P/m)^2$, where m is the mass of the unit charge. For example, a continuous global $U(1)$ ‘baryon number’ symmetry, must be explicitly broken by gravity, at least down to a Z_N subgroup, with $N \lesssim (M_P/m_b)^2$, where m_b is the baryon mass. The same constraint applies to any discrete gauge symmetry, as well as to other quantum-mechanically-detectable black hole charges that are associated with the massive quantum hair of the black hole. We show that the gravitationally-coupled N -species sector that solves the gauge hierarchy problem, should be probed by LHC.

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1 Introduction

The essence of the hierarchy problem is an inexplicably large separation between the weak and the Planck (M_P) scales, or equivalently between M_P and the Higgs mass. The latter, is quadratically sensitive to the cut-off of the theory, and this is the source of the problem. Either the cut-off is very high (say $\sim M_P$), and then one is left with the question, what stabilizes the Higgs mass? Or, the cut-off is not far from the weak scale, but then one has to explain why gravity is so weak. In both cases, the solution should come up in the form of an UV-insensitive large number, that sets the hierarchy between the scales. For example, in supersymmetry, this number is the ratio of the the Plack mass to the supersymmetry breaking scale, whereas in Large Extra Dimensions scenarion [11], the hierarchy is set by the large volume of the extra space (in fundamental Planck units).

In the present paper, we shall discuss a different approach in which a large UV-stable number that provides the hierarchy between the Planck mass and a low energy scale Λ , is the the number of the quantum field species, N .

We shall show that in the theories in which the number of species at the scale Λ is $N \gg 1$, the Plack mass must satisfy

$$M_P^2 \gtrsim N \Lambda^2, \quad (1.1)$$

up to a factor that scales as $\sim \ln(N)$ with large N . We shall provide the two types of arguments supporting this bound. The first one is an already existing perturbative argument, which shows that having $M_P^2 \ll N \Lambda^2$ for large N is technically unnatural.

The second one is an exact non-perturbative argument, which shows that having a Planck mass that violates the bound (1.1) is inconsistent with the black hole physics. In the other words, we are lead to the conclusion that the large number of species automatically weakens gravity by a $1/N$ -factor!

Then, the following ‘cheap’ solution to the hierarchy problem emerges. Postulate the existence of $N \sim 10^{32}$ quantum field species beyond the Standard Model. For instance, N copies of the Standard Model related by the permutation symmetry. Although, a low energy observer from each Standard Model replica would be puzzled by the smallness of the weak scale versus the Planck mass, the hierarchy would be guaranteed by the consistency of the theory with the black hole physics.

The proof of the bound (1.1) delivers an important byproduct, which relates the periodicity of the conserved quantum numbers, to the allowed values of the Planck mass, in a model-independent way. In particular, any conserved quantum number Q , which is not associated with a long-range classically-observable force, and is carried by the state(s) of mass Λ , can have a maximal periodicity N given by (1.1).

For example, this constraint automatically applies to the gauged discrete Z_N -symmetries, as well as to any exact subgroup of the continuous global symmetries, in general. It also automatically results into the quantization of the black hole quantum charge associated with a massive quantum hair. Two types of the quantum hair are of our interest.

The fact that the black holes can carry a quantum mechanical hair under the discrete gauge symmetries, was discovered some time ago by Krauss and Wilczek [1]. Such a quantum hair results whenever a continuous gauge group is spontaneously broken (Higgsed) down to a discrete subgroup at an arbitrarily high scale. Because of its Z_N -nature, such a hair automatically falls within our constraint.

Interestingly, the above bound (1.1) also constraints a different type of the black hole quantum hair. In [2], it was shown that the black holes can carry a quantum-mechanical hair under the massive gauge fields of an arbitrary integer spin, such as spin-2, or higher. Naively, the corresponding charge is continuous and can take an arbitrary periodicity. However, our arguments show that this is not true. Since just like the Z_N -case, the massive spin-2 (or higher) hair is not supported by any classically-detectable long-range field, the corresponding charge automatically falls within the validity range of our proof, and must be N -periodic, with N satisfying the bound (1.1). Thus, the quantum-mechanically detectable black hole charges always have limited periodicity bounded by (1.1).

Thus, the number of species in the large N solution of the hierarchy problem, can equally be replaced by a large N -cycle of a Z_N gauge symmetry, or with a large N -periodicity of the black hole charge under some massive quantum hair.

As one of the central points, we shall discuss a phenomenologically-interesting observation, that a hidden sector with N -species, that guaranties the hierarchy between the Planck mass and the TeV scale, must be probed in the Standard Model particle collisions at energies \sim TeV.

Finally, the String Theory constructions often involve many species of quantum fields at low energies. This fact may be a blessing for stabilizing the various scales in String Theory.

For example, one can ask, whether a large number of species could be a viable way of explaining the hierarchy between the Planck mass and the Grand Unified Theory (GUT) scales, since the realistic theories often incorporate large number of states at the GUT scale.

2 Perturbative Renormalization of the Planck Mass

We start with the perturbative argument first. This was suggested in [3] as a possible many-species solution to the hierarchy problem in the context of the brane world gravity [4]. The idea being that on a 3-brane populated by the N four-dimensional species, the effective Planck

mass should scale as N , even if the cutoff (which can be either the Standard Model scale or the fundamental high-dimensional Planck mass) is fixed. Our perturbative argument is exactly the same, although we wish to abstract from any particular high-dimensional realization and reduce the idea to its bare essentials in four-dimensions.

The point then is the following. Imagine N species of quantum fields, with masses at the scale Λ , coupled to gravity. Each of these species will contribute into the renormalization of the Planck mass (equivalently, the graviton wave-function renormalization) the factor $\sim \Lambda^2$ [5]. Neglecting the accidental cancellations, this has to be multiplied by the number of species, and as a result the effective contribution to the Planck mass is $\sim N\Lambda^2$. In the other words, the perturbative renormalization arguments suggest that in the presence of N -species gravity must be weakened by $1/N$.

3 The Black Hole Proof

3.1 The Leading Order Relation

We shall now discuss a non-perturbative argument, based on the black hole physics, that proves the bound (1.1). Consider N species of the bosonic quantum fields Φ_j , $j = 1, 2, \dots, N$, of mass Λ . We shall first assume that the system is invariant under an *exact* discrete $Z_2^N \equiv Z_2^{(1)} \times Z_2^{(2)} \times \dots \times Z_2^{(N)}$ symmetry, under the independent sign flips of the fields. That is, under any given $Z_2^{(j)}$ -factor only one particular field changes the sign, $\Phi_j \rightarrow -\Phi_j$, whereas all the other fields are invariant.

We shall now prove that in such a case the Planck mass must satisfy the bound (1.1).

In order to prove the relation (1.1) we can perform the following thought experiment. Taking an arbitrarily large number of N -species particles, we can prepare an arbitrarily large black hole. This black hole will carry the information about the amount of the conserved charge carried by the particles. In order to avoid entering the discussions on the black hole information loss issues, it is useful to think of these Z_2 -s as the gauged discrete symmetries [1]. The information about the absorbed charge then can be monitored by the Aharonov-Bohm effect at infinity, using the probe Z_2 -cosmic strings [6], and cannot be lost.

Because the conserved quantum number is Z_2^N , we can store maximum N units of the charge in such a black hole. For this we will need N particles, each belonging to a different species. Any further increase of the number of the initial particles, will not increase the amount of the conserved discrete charge stored by the black hole. Thus, we shall focus on a minimal size black hole carrying the maximum possible discrete charge.

The mass of such a black hole is

$$M_{BH} = N\Lambda \quad (3.1)$$

Because of the conservation, the information about the Z_2^N -charge hosted by the black hole, must be revealed after its evaporation. For a black hole of the Hawking temperature T_H , the probability of the emission of a heavy particle of mass $\Lambda \gg T_H$ is exponentially suppressed by a Boltzmann factor $\sim e^{-\frac{\Lambda}{T_H}}$. Thus, our black hole with N units of the Z_2^N -charge, can start emitting N -species particles, only after its temperature drops to $T_H \sim \Lambda$. At this point, the mass of the black hole is $M_{BH}^* \sim \frac{M_P^2}{\Lambda}$. Starting from this moment, the black hole can start revealing back the stored charge, in form of the N -species particles. However, by conservation of energy, the maximum number of particles that can be emitted by the black hole is

$$n_{max} \sim \frac{M_P^2}{\Lambda^2}. \quad (3.2)$$

These states should carry the same Z_2^N -charge as the original N -particles. Thus, $n_{max} = N$, which proves the equation (1.1).

In the other words, the key point of the proof, is that the amount of the maximal discrete charge that is stored in the initial black hole scales as N , but the temperature at which the black hole starts giving back this charge essentially *does not* scale with N . Hence the only way to avoid inconsistency is the scaling of the Planck mass² as $\sim N$.

3.2 $\ln N$ -corrections

We should stress that the limit (1.1) must be understood to leading order in large N , up to corrections that behave as $\ln N$. To see this let us go through the black hole evaporation process more carefully. The rate of production of a particle of mass Λ for a black hole of the Hawking temperature $T_H \ll \Lambda$ is

$$\Gamma \sim T_H e^{-\frac{\Lambda}{T_H}}. \quad (3.3)$$

The total number of such particles from all the N -species produced during a time t_* is

$$n(t_*) \sim N \int_0^{t_*} T_H e^{-\frac{\Lambda}{T_H}} dt. \quad (3.4)$$

Noting that the cooling rate of the black hole (in the other species of mass $\ll T_H$) is

$$\frac{dM_{BH}}{dt} \sim -T_H^2, \quad (3.5)$$

and using the relation between the black hole mass and the temperature $T_H \sim M_P^2/M_{BH}$, we can re-express (3.6) as an integral over the temperature

$$n(t_*) \sim N \int_{T_{in}}^{T_*} dT_H \frac{M_P^2}{T_H^3} e^{-\frac{\Lambda}{T_H}}, \quad (3.6)$$

where T_{in} is the initial temperature, which for the minimal black hole of interest is $T_{in} \sim \frac{M_P^2}{N\Lambda}$. Since we are proving the bound (1.1), it is enough to show that the opposite assumption leads us to the contradiction. So assume that M_P could violate the bound (1.1) so that we could have $M_P^2 \ll N\Lambda^2$. Then $T_{in} \ll \Lambda$, and the initial emission of N -species is exponentially suppressed. We now integrate (3.6) to T_* defined by the condition that $n(t_*) \lesssim 1$. In the other words, we choose T_* to be a temperature defined by the requirement that before reaching it the black hole managed to emit only of order one number of particles from the entire variety of the N -species.

Obviously, we have $T_* \sim \Lambda / \ln \left(\frac{NM_P^2}{\Lambda^2} \right)$, or the corresponding mass M_{BH}^* is

$$M_{BH}^* \sim \frac{M_P^2}{\Lambda} \ln \left(\frac{NM_P^2}{\Lambda^2} \right), \quad (3.7)$$

which corrects the bound (1.1) by a factor $\sim \ln \frac{NM_P^2}{\Lambda^2}$, or equivalently by a factor $\sim \ln N^2$.

4 Bound on M_P for the Unstable N -Species

We shall now repeat our reasoning for the case in which N -species are not exactly stable and can decay into some lighter states, e.g., such as the light Standard Model particles. Let the lifetime of the species be τ_N . Our proof of the equation (1.1) then can still be applied as long as this lifetime is longer than the lifetime of the minimal black hole carrying the N -units of the Z_2^N charge. The lifetime of such a black hole is

$$\tau_{BH} \sim \frac{M_{BH}^3}{M_P^4} = \frac{N^3 \Lambda^3}{M_P^4}. \quad (4.1)$$

Thus, as long as

$$\tau_N \gtrsim \frac{N^3 \Lambda^3}{M_P^4}, \quad (4.2)$$

our proof should hold, and we have a limit on M_P given by (1.1). Substituting this in to the (4.2), we get a bound on τ_N in terms of N and Λ , for which our proof is still applicable

$$\tau_N \gtrsim \frac{N}{\Lambda}. \quad (4.3)$$

This expression is interesting because of the following reason. First, observe that the lower bound of this expression is achieved if N -species decay into the lighter states through the operators suppressed by the powers of the scale $M \equiv \sqrt{N}\Lambda$, which, according to (1.1), saturates the lower bound on the Planck mass. Secondly, the same scale M is independently suggested by the perturbative arguments as the minimal scale suppressing the strength of the interactions of any representative of the system of N inter-coupled species, because of the N -fold contribution into the renormalization of their wave-functions.

5 Black Hole Constraints on the Conserved Quantum Numbers

Our analysis has immediate implication for the conserved charges of arbitrary sort that are not characterized by any locally-observable long-range (massless) fields. For example, such are the charges conserved due to the gauged discrete symmetries. Our proof implies that any exactly conserved quantum number of the above sort (call it Q) must be defined modulo (be periodic) N_{max} , with

$$N_{max} \lesssim \left(\frac{M_P}{m} \right)^2, \quad (5.1)$$

where m is the mass of a particle carrying one unit of the Q -charge. We shall prove this bound for the simplest case when there is a single species carrying the charge Q , generalization to more complicated cases is straightforward. In the latter situation when there are multiple species carrying the different amount of the Q charge, what counts is the mass to charge ratio.

We can again perform the similar thought experiment as for N -species, but instead of putting together N_{in} particles from different species, we shall form a black hole by putting together N_{in} units of the Q -charge. Since each unit carries a mass m the minimal mass of such a black hole is $M_{BH} = N_{in} m$. Since we wish to prove that Q must have a limited periodicity of N_{max} given by (5.1), it is enough to show that the opposite assumption leads us into a contradiction.

Thus, we assume that the charge Q has an unlimited periodicity. That is, can be arbitrarily large in terms of the elementary charge. Then, by increasing N_{in} , we can store an arbitrarily large amount of the Q charge into the black hole. But then again, the black hole cannot return back any significant amount of the Q -charge up until it evaporates down to the mass $M_{BH}^* \sim \frac{M_P^2}{m}$, at which point its Hawking temperature becomes comparable to the mass of an elementary Q -quanta.

After this moment, however, by conservation of energy, the black hole can only give back $N_{max} \sim M_P^2/m^2$ units of the Q -charge, which contradicts to the initial condition that we could make initial Q -charge of the black hole arbitrarily large, by increasing N_{in} unbounded. The only way the story can be made consistent, is if Q -charge is defined modulo N_{max} . In such a case irrespective of the initial number of charged quanta, the total charge of the black hole cannot exceed N_{max} , and inconsistency is avoided.

One can come up with many situations for which the above proof puts important constraints, and we shall now briefly review some of them.

5.1 Consistency Relation Between the Planck Mass and the Size of a Z_N Gauge Symmetry

For example consider a single complex scalar field Φ of mass m transforming under a discrete gauge symmetry Z_N

$$\Phi \rightarrow e^{i\frac{2\pi}{N}} \Phi, \quad (5.2)$$

Then, the maximum value of N is limited by (5.1).

We can turn the argument around, and say that the presence of a Z_N -symmetry at any scale Λ , implies the existence of the hierarchy between the scale Λ and the Planck mass given by (1.1), in which N has to be understood, as the periodicity of the Z_N group.

5.2 Maximal Allowed Z_N -subgroups, of the Continuous Global Symmetries

Another important implication is for the continuous global symmetries. Consider, for example, a single scalar field Φ of mass m transforming under a global $U(1)$ ‘baryon number’ symmetry. If $U(1)$ were exact, there would be a conserved baryon number Q , with unlimited periodicity. According to the above proof, however, this is impossible, and Q can only be defined modulo N_{max} given by (5.1). This means that any would be continuous global symmetry must be inevitably broken by the gravitational effects, at least down to the $Z_{N_{max}}$ subgroup. That is, in the effective low energy theory, gravity should generate $U(1)$ -violating operators of the form

$$\Phi^N + \Phi^{*N} \quad (5.3)$$

with $N \leq N_{max}$.

5.3 Quantization of the Black Hole Massive Quantum Hair

Finally, the bound (5.1) also implies the quantization of the black hole quantum hair under the massive integer spin gauge fields.

It was pointed out recently [2] that in the presence of spin-2 or higher integer-spin massive fields, the black holes may be endowed with the quantum hair under these fields. The field configuration is locally pure gauge, and because of this it is classically unobservable in full agreement with the standard no-hair arguments [7–9] and in particular the classical result by Bekenstein [7]. However, the gauge field configuration has a global topological structure, which makes it observable quantum-mechanically. In particular, such a hair can be detected at infinity by the Aharonov-Bohm [10] type experiment.

The essence of this quantum hair can be understood in the following way (see [2] for the details). The massive high integer spin fields (e.g., spin-2) include a spin-1 "longitudinal" component \mathcal{A}_μ , which plays the role of a Goldstone-Stückelberg field for maintaining the gauge invariance in the presence of the mass. Under the gauge transformation, \mathcal{A}_μ shifts as

$$\mathcal{A}_\mu \rightarrow \mathcal{A}_\mu - \xi_\mu, \quad (5.4)$$

where ξ_μ is an arbitrary regular vector. A black hole with a quantum hair corresponds to the solution for which the spin-1 component has a form of Dirac's magnetic monopole [2]

$$F_{ij} = \mu \epsilon_{ijk} \frac{x^k}{r^3}, \quad F_{0j} = 0, \quad (5.5)$$

where $F_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$, x_j are space coordinates and μ is the quantum charge. However, unlike the would be Dirac's magnetic monopole, this configuration is locally pure gauge, since \mathcal{A}_μ is exactly compensated by the other components of the massive high spin field, so that the full field is identically zero. This is why it cannot be probed classically by any local experiment. However, it can be probed quantum mechanically provided there are boundary terms of the following form

$$q \int dX^\mu \wedge dX^\nu F_{\mu\nu}. \quad (5.6)$$

This term describes a boundary coupling of the Stückelberg \mathcal{A}_μ to a test string, where X^μ are the string target space coordinates. q is a constant. Since we are interested in large distance effects, the microscopic nature of the string is unimportant. Because (5.6) is a boundary term, it is not affecting dynamics at the classical level, but quantum mechanically can lead to an observable effect in the presence of the quantum hair (5.5). The string can detect the quantum hair of the black hole, by the Aharonov-Bohm experiment, in which the string loop lassoes the black hole. The phase shift resulting from such an experiment is

$$\text{phase shift} = 4\pi\mu q, \quad (5.7)$$

and is observable as long as $\mu q \neq n/2$. Thus, black holes can have an additional locally-unobservable quantum charge $Q \equiv 2\mu q$. We shall now show that our black hole arguments impose the following quantization condition on Q .

Let the mass of a black hole (or a particle), that carries an elementary unit of the quantum charge Q , be m . Naively, this quantum charge Q can be arbitrarily small. Then, because Q causes no locally-observable long-range field, there is no obstruction in putting together an arbitrary number N of the unit charges and producing a black hole that would store NQ quantum charge, which should be given back after evaporation. If Q could be arbitrarily small,

then N could be arbitrarily large. However, according to our proof this is impossible. Which implies inevitable periodicity of Q set by (5.1). Or in the other words quantization of Q in the units of $\frac{1}{N_{max}}$.

6 Implications for the Hierarchy Problem and LHC

From the above analysis, there emerge the two possible solutions to the hierarchy problem.

The first approach is to guarantee weakness of gravity by postulating a discrete symmetry, with a huge periodicity, around the TeV scale.

For instance, we may postulate that beyond the Standard Model there is an additional complex scalar Φ with a \sim TeV mass, that transforms under some Z_N -symmetry with $N = 10^{32}$. Then, this fact would automatically imply the needed weakness of gravity.

Another approach is to simply postulate that there are additional 10^{32} species on top of the standard model, or even 10^{32} replicas of the Standard Model, all related by a permutation symmetry.

Although seemingly different, the connection between these two approaches may be stronger than what one would naively guess. The reason is, that in many instances generation of large discrete symmetry groups implies the existence of many species in the underlying theory, or else we may violate some conditions of our theorem, and weakness of gravity will no longer be guaranteed. To illustrate this point, let us consider an example in which one wants to generate an effective Z_N -symmetry, with $N \sim 10^{32}$, from a renormalizable theory with order one couplings. We can achieve this by introducing a set of n scalar fields Φ_k , $k = 1, 2, \dots, n$, with the following sequence of couplings

$$\Phi_1^3 + \Phi_1^* \Phi_2^3 + \Phi_2^* \Phi_3^3 + \dots \Phi_k^* \Phi_{k+1}^3 + \dots + \Phi_{n-1}^* \Phi_n^3, \quad (6.1)$$

where under Z_{3^n} ,

$$\Phi_k \rightarrow e^{i \frac{\pi}{3^k}} \Phi_k. \quad (6.2)$$

Integrating out all $n - 1$ fields, the effective coupling for Φ_n will be

$$\Phi_n^{3^n} + h.c., \quad (6.3)$$

which exhibits, Z_{3^n} -invariance. The above construction leaves an impression that we have managed to generate a discrete Z_N -symmetry with exponentially large $N = 3^n$, by introducing just n fields. This is true, however there is the following caveat as far as the solution of the hierarchy problem is concerned. If we don't want to have any input hierarchy of scales, the masses of all the integrated out fields must not be much above Λ . However, in the latter case,

the condition of our theorem is violated, since fields with $k < n$ will inevitably have much larger Z_{3^n} -charge-to-mass ratio than Φ_n . A black hole prepared with an arbitrarily large number of Φ_n fields can get rid of its Z_{3^n} -charge by emitting just $\sim n$ number of Φ_k -quanta with needed values of k , without implying large M_P/Λ hierarchy.

The lesson from the above example is, that generation of large discrete symmetries without violating any of the conditions of the bound (1.1), may require either an unnaturally small coupling, or the large number of fields. For example, N fields with exact permutation symmetry, would do such a job.

One way or the other, we are lead to the conclusion that the large N solution of the hierarchy problem (whether N refers to the number of species or to the periodicity of the Z_N symmetry) implies the existence of a sector beyond the Standard Model with enormous number of quantum fields. This fact is the key for the testability of the above proposal. We shall now argue, that under very reasonable assumptions this sector will be probed by LHC.

Since the N -sector states are singlets under the standard model gauge group, their couplings to the SM sector must be through the effective non-renormalizable interactions. The exceptions are the spin-1 and spin-0 fields. The former could couple, for instance, to baryon or lepton number currents, and the latter could have a renormalizable couplings with the Standard Model Higgs doublet. Also, gauge-neutral neutrinos from the different species could also mix.

Renormalizable or not, the strength of the N -species interaction with the Standard Model fields must be suppressed by the powers of the Planck mass, provided most of them couple to our sector and to each other. In the opposite case one has to explain why the standard model fields themselves are not weakly coupled, since the N -sector loops would lead to large wave function renormalization of the Standard Model fields, just like they do for the graviton.

Hence, we expect the N -sector species to couple with the standard model through the effective operators of the form

$$\sum_i \frac{\Phi_i}{M_i} T_\mu^\mu + \sum_i \frac{\Phi_{\mu\nu}^{(i)}}{\bar{M}_i} T^{\mu\nu} + \dots \quad (6.4)$$

Here we gave the examples of spin-0 and spin-2 species respectively, and $T_{\mu\nu}$ is the energy momentum tensor of the Standard Model fields. According to our arguments, scales M_i, \bar{M}_i must scale as $\sim N$, and thus, be of the order of M_P . The N -species then will be produced in the collision of SM particles, and in particular at LHC. Although the production rate of any particular species is gravitationally suppressed, their number overcompensates. The production rate of all the species above their mass, in some collision process at typical energy E , is

$$\Gamma_{total} \sim \frac{E^3}{M_P^2} N \sim \frac{E^3}{\Lambda^2}, \quad (6.5)$$

where in the last expression we have used the bound (1.1). This expression, is the part of the general rule, which states the following. By allowing the natural strength (which is also a maximal allowed strength) coupling to the N -species that maintain the hierarchy between the Planck mass and any scale $\Lambda \ll M_P$, their production rate becomes important at the scale Λ . Since in our case $\Lambda \sim \text{TeV}$, the relevance for the LHC automatically follows.

We wish to point out some resemblance between (6.5) and the production rate of the Kaluza-Klein gravitons in Large Extra Dimensional model [11]. In the light of our discussion, this is not at all surprising, since Large Extra Dimensions solution of the hierarchy problem, can be regarded as a particular example of the large N -solution, due to the fact that it also includes $\sim 10^{32}$ Kaluza-Klein gravitons of mass $\sim \text{TeV}$. Thus, even if one knew nothing about the high-dimensional Gauss law and the relation between the fundamental and four-dimensional Planck masses, the solution of the hierarchy problem, in the light of the presented black hole arguments, becomes obvious the moment one realizes that there are 10^{32} Kaluza-Klein species around the TeV scale.

Finally, some open questions are in order. The natural one to ask is, what is the connection between the bound (1.1) Bekenstein-Hawking entropy.

It also would be interesting to explore the connection between the work [12] and the arguments presented here.

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